MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

ETN3096 – DIGITAL SIGNAL PROCESSING

(CE, EE, LE, NE, OPE, TE)

13 MARCH 2018
2:30 PM – 4:30 PM
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 12 pages (including this cover page) with 4 Questions only and an appendix.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please print all your answers in the Answer Booklet provided.

Question 1

(a) Figure 1(a) shows a linear time-invariant (LTI) system consisting of two sub-systems connected in parallel.

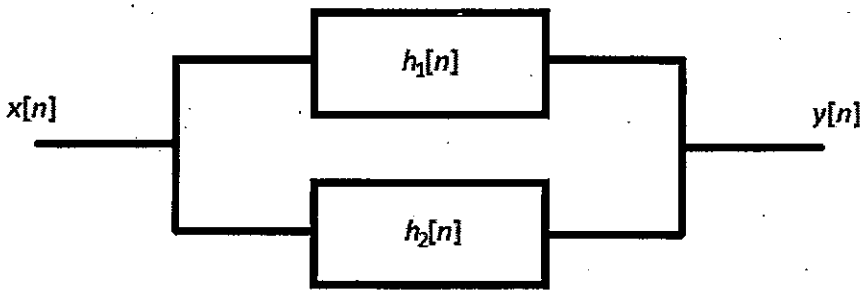


Figure 1(a): LTI system output $y[n]$, whereas $x[n]$ denotes the input, and $h_1[n]$ and $h_2[n]$ denotes sub-systems 1 and 2 respectively.

(i) Give two applications of digital signal processing (DSP). [2 marks]

(ii) Give the expression for the system output $y[n]$, in terms of $x[n]$, $h_1[n]$ and $h_2[n]$.

[2 marks]

(iii) If $h_1[n] = \{0,\ 2,\ \underline{1},\ 4,\ 3,\ 1\}$ (where the underline denotes sample at $n = 0$), and $h_2[n] = \delta[n] - 3\delta[n-1] - 5\delta[n-2]$, calculate $y[n]$ if $x[n] = \{1,\ 0,\ \underline{1},\ 1\}$. Present your answer in graphical form.

[6 marks]

(b) For the following system:

$$y[n] = x[n]\cos(0.2\pi n)$$

It is given that $x[n]$ is bounded, i.e., $|x[n]| \leq X$. Determine if the system is:

(i) Linear
(ii) Time-invariant
(iii) Causal

Justify your answers using proofs where necessary.

[6 marks]

Continued ...

Question 1 (continued)

(c) Express the signal shown in Figure 1(b) using the basic sequences $\delta[n]$ and $u[n]$.

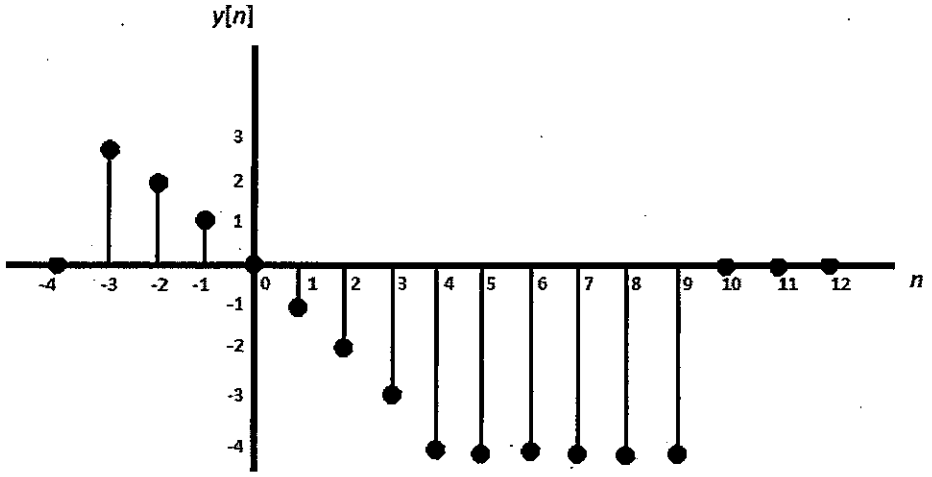


Figure 1(b): A finite discrete signal.

[3 marks]

(d) A continuous signal, $x_c(t)$, is given by:

$$x_c(t) = 2\sin(2\pi 512t) + 3.5\sin(2\pi 1024t)$$

(i) Determine the minimum sampling frequency for $x_c(t)$. Subsequently, obtain the expression of the sampled signal $x[n]$.

[2 marks]

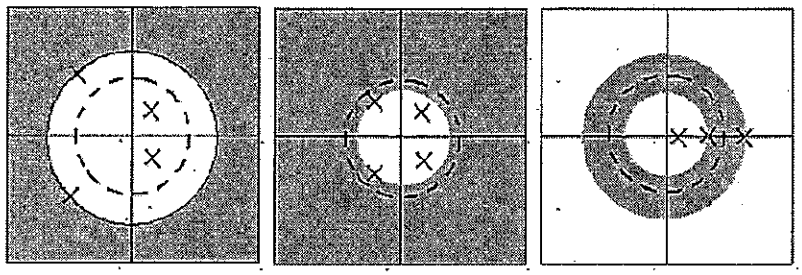
(ii) With the help of a diagram that depicts the signal in frequency domain, explain the consequences of sampling the signal with a sampling period of 4 ms.

[4 marks]

Continued ...

Question 2

(a) Figure 2 shows the pole-zero plots of three systems.



(i) (ii) (iii)

Figure 2(i) to (iii): Dashed lines denote the *unit circle*, the pole locations are marked by 'x's, the shaded region denotes the region of convergence (ROC).

(i) For each diagram shown in Figure 2, determine whether or not the system is realizable. Justify your answers.

[6 marks]

(ii) If the pole locations for the system shown in Figure 2(iii) are located at **0.2, 0.7** and **1.3**, derive the expression for the system function $H(z)$ and subsequently, its frequency response $h[n]$. Assume arbitrary coefficients A, B and C for each term.

[6 marks]

(b) A signal $x[n]$ is defined by the following finite sequence

$$x[n] = \delta[n] + 2\delta[n-2] + \delta[n-3]$$

(i) Compute the 4-point Discrete Fourier Transform (DFT) of $x[n]$

[6 marks]

(ii) Let $w[n] = x[\langle n\text{-}2\rangle_4]$ be the circularly shifted signal of $x[n]$. Plot the signal $w[n]$ and find its DFT.

[7 marks]

Continued ...

Question 3

(a) You are required to design a 3-tap FIR **highpass** filter with a cutoff frequency of 1,500 Hz and a sampling rate of 8,000 sample/s. Using a rectangular window function, determine the followings for the filter:

(i) Filter coefficients [13 marks]
(ii) Transfer function [2 marks]
(iii) Difference equation. [2 marks]

(b) Design a first-order lowpass digital Chebyshev filter with a cutoff frequency of 1.5 kHz and 1 dB ripple on the passband using a sampling frequency of 8 kHz. Determine the transfer function and difference equation.

[16 marks]

Question 4

An IIR filter has been designed with a transfer function $H(z)$ defined by a constant gain of two, **four poles** at 0, 0.1, -0.5, -0.4, and **one zero** at -2.

(i) Derive the transfer function $H(z)$.

[3 Marks]

(ii) Implement the filter as a cascade structure where each block is **first order**. Draw the structure.

[8 Marks]

(iii) Implement the cascade structure for $H(z)$ by merging the two poles (-0.5 and -0.4) with the zero (-2) into a single **second order** block. Draw the structure.

[6 Marks]

Continued ...

APPENDIX

The z-transform

Properties of the z-transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n] + by[n]$	$aX(z) + bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n-m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n] * y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common z-transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n-n_0]$	z^{-n_0}	Possibly $\forall z$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$

Discrete Time Fourier Transform and its Inverse

$$X(\Omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\Omega n}$$

$$x(n) = \frac{1}{2\pi}\int_0^{2\pi} X(\Omega)e^{j\Omega n}d\Omega$$

Summary of Ideal Impulse Responses for Standard FIR Filters

Filter Type	Ideal Impulse Response $h(n)$
Lowpass:	$h(n) = \begin{cases} \frac{\Omega_c}{\pi}, & n = 0 \\ \frac{\sin(\Omega_c n)}{n\pi}, & n \neq 0 \end{cases}$
Highpass:	$h(n) = \begin{cases} \frac{\pi - \Omega_c}{\pi}, & n = 0 \\ \frac{-\sin(\Omega_c n)}{n\pi}, & n \neq 0 \end{cases}$
Bandpass:	$h(n) = \begin{cases} \frac{\Omega_H - \Omega_L}{\pi}, & n = 0 \\ \frac{\sin(\Omega_H n) - \sin(\Omega_L n)}{n\pi}, & n \neq 0 \end{cases}$
Bandstop:	$h(n) = \begin{cases} \frac{\pi - (\Omega_H - \Omega_L)}{\pi}, & n = 0 \\ \frac{-\sin(\Omega_H n) + \sin(\Omega_L n)}{n\pi}, & n \neq 0 \end{cases}$

FIR Filter Length Estimation Using Window Functions

Window Type	Window Function $w(n), -M \le n \le M$
Rectangular	1
Hanning	$0.5 + 0.5\cos\left(\frac{n\pi}{M}\right)$
Hamming	$0.54 + 0.46\cos\left(\frac{n\pi}{M}\right)$
Blackman	$0.42 + 0.5\cos\left(\frac{n\pi}{M}\right) + 0.08\cos\left(\frac{2n\pi}{M}\right)$

FIR Filter Length Estimation Using Window Functions

Normalized Transition Width $\Delta f = \frac{\lvert f_{stop} - f_{pass} \rvert}{f_{sampling}}$			
Type of Window	**Window Length N**	**Stopband Attenuation, $20\log_{10}\delta_s$ (dB)**	**Passband Ripple**
Rectangular	$N = 0.9/\Delta f$	21	0.7416
Hanning	$N = 3.1/\Delta f$	44	0.0546
Hamming	$N = 3.3/\Delta f$	53	0.0194
Blackman	$N = 5.5/\Delta f$	74	0.0017

Bilinear Transformation (BLT)

1. Frequency prewarping
Let ω_a denote the analog frequency marked on the $j\omega$-axis on the s-plane, and ω_d denote the digital frequency marked on the unit circle in the z-plane.

For the lowpass filter and highpass filter:

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right)$$

For the bandpass filter and bandstop filter:

$$\omega_{al} = \frac{2}{T}\tan\left(\frac{\omega_l T}{2}\right),\ \omega_{ah} = \frac{2}{T}\tan\left(\frac{\omega_h T}{2}\right)$$

and $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, W = \omega_{ah} - \omega_{al}$.

2. Prototype transformation using the lowpass prototype $H_P(s)$
From lowpass to lowpass: $H(s) = H_P(s)\big|_{s=\frac{s}{\omega_a}}$

From lowpass to highpass: $H(s) = H_P(s)\big|_{s=\frac{\omega_a}{s}}$

From lowpass to bandpass: $H(s) = H_P(s)\big|_{s=\frac{s^2+\omega_0^2}{sW}}$

From lowpass to bandstop: $H(s) = H_P(s)\big|_{s=\frac{sW}{s^2+\omega_0^2}}$

where ω_a denotes the analog frequency, $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, W = \omega_{ah} - \omega_{al}$.

3. Substitute the BLT to obtain the digital filter

$$H(z) = H(s)\big|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$

Conversion from Analog Filter Specifications to Lowpass Prototype Specifications

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$\nu_s = \dfrac{\omega_{as}}{\omega_{ap}}$
Highpass: ω_{ap}, ω_{as}	$\nu_s = \dfrac{\omega_{ap}}{\omega_{as}}$
Bandpass: $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$ $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_s = \dfrac{\omega_{ash} - \omega_{asl}}{\omega_{aph} - \omega_{apl}}$
Bandstop: $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$ $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_s = \dfrac{\omega_{aph} - \omega_{apl}}{\omega_{ash} - \omega_{asl}}$

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge

ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband

ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband

ω_0, geometric center frequency

Digital Butterworth and Chebyshev Filter Designs

With the given passband ripple A_p dB at the normalized passband frequency edge $\nu_p = 1$, and the stopband attenuation A_s dB at the normalized stopband frequency edge ν_s, we determine the lowpass prototype order as

$$\varepsilon^2 = 10^{0.1A_p} - 1$$

The **Butterworth** lowpass prototype with order n is given as

$$n \geq \frac{\log_{10}\left(\dfrac{10^{0.1A_s} - 1}{\varepsilon^2}\right)}{|2\log_{10}(\nu_s)|}$$

where ε is the absolute ripple specification.

The **Chebyshev** lowpass prototype with order n is given as

$$n \geq \frac{\cosh^{-1}\left(\sqrt{\dfrac{10^{0.1A_s} - 1}{\varepsilon^2}}\right)}{\cosh^{-1}(\nu_s)}$$

where $\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$.

3-dB Butterworth Lowpass Prototype Transfer Functions ($\varepsilon = 1$)

n	$H_p(s)$
1	$\frac{1}{s+1}$
2	$\frac{1}{s^2+1.4142s+1}$
3	$\frac{1}{s^3+2s^2+2s+1}$

Chebyshev Lowpass Prototype Transfer Functions with 1 dB Ripple ($\varepsilon = 0.5088$)

n	$H_p(s)$
1	$\frac{1.9652}{s+1.9652}$
2	$\frac{0.9826}{s^2+1.0977s+1.1025}$
3	$\frac{0.4913}{s^3+0.9883s^2+1.2384s+0.4913}$

CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

Discrete-time Fourier Analysis

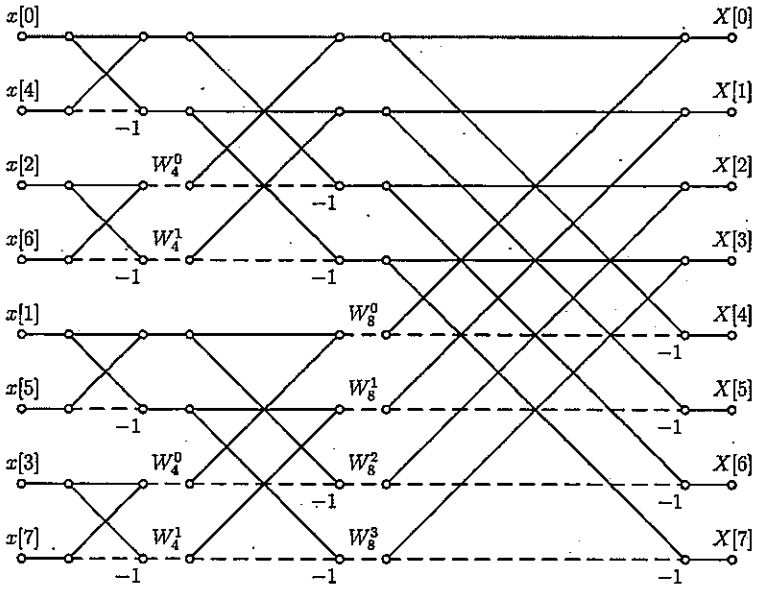
The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

Properties of the DFT

Property	$x[n]$	$X[k]$
Linearity	$A_1 x_1[n] + A_2 x_2[n]$	$A_1 X_1[k] + A_2 X_2[k]$
Time shifting	$x[\langle n - n_0 \rangle_N]$	$X[k] W_N^{kn_0}$
Frequency shifting	$x[n] W_N^{-k_0 n}$	$X[\langle k - k_0 \rangle_N]$
Time reversal	$x[\langle -n \rangle_N]$	$X[\langle -k \rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k \rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k] Y[k]$
Modulation	$N x[n] y[n]$	$X[k] \circledast Y[k]$

The decimation-in-time FFT



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